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### A parametric strategy for free-form glass structures using quadrilateral planar facets

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### Abstract

The design and construction of free-form glass roofing structures is generally accomplished through the use of either planar triangular glass facets or curved (formed) glass panes. This paper describes ongoing research on the constructability of such structures using planar quadrilateral glass facets for the Jerusalem Museum of Tolerance project by Gehry Partners, in collaboration with Schlaich Bergermann & Partners, engineers. The challenge here lies not only in the development of a geometric strategy for generating quadrilateral planar facet solutions, but also in the fact that said solutions must closely match the designs created initially in physical model form by the architects.

We describe a simple but robust geometric method for achieving the structure by incorporating the necessary geometric principles into a computational parametric framework using the CATIA Version 5 system. This generative system consists of a hierarchical set of geometric 'control elements' that drive the design toward constructible configurations. Optimization techniques for approximating the generated structural shape to the original created by the designers are also described. The paper presents the underlying geometric principles to the strategy and the resulting computational approach. © 2003 Published by Elsevier B.V.

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### 1. Introduction

The design for the Jerusalem Museum of Tolerance (referred to in this paper as MOT), a project of the Simon Wiesenthal Center, is currently under development at Gehry Partners. This voluminous complex is a multifunctional group of building components, each of which is given unique design character within the context of the overall design (Fig. 1). A major component of the project is a series of large, freeform glass structures that are the subject of discussion in this paper. Seven glass surface structures are used, including some functioning as walls. The two largest segments cover a large space formed by a curved Museum building and two other components, the Grand Hall and the Research Center, and were the subjects of the initial investigations described in this paper.

Resolving these glass structures is a twofold problem: first, it is a problem of computational geometry and resulting implications for fabrication; second, it is a problem of user interaction and design methods. The

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Fig. 1. The Museum of Tolerance Project, aerial view.

latter problem is of particular importance, because Gehry Partners has developed a very specific way of working with regard to how designs are created and developed, and how the computer supports this process. Finding a viable geometrical solution mechanism that fits into this framework is paramount in ensuring that the generated forms reflect the original design intention.

### 2. From physical models to digital mockups

Over the past 10 years, Gehry Partners (formerly Frank O. Gehry and Associates) has developed a set of unique working methods centered on the concept of the "Master Model". In the context of Frank Gehry's work, the firm uses the computer as a design development and design production tool: in other words,



Fig. 2. The Museum of Tolerance CATIA V5 Master Model.



Fig. 3. Data acquisition from physical models.

the computer is used to capture a formal design intention and make it constructible (Fig. 2).

Within the Gehry studio, the designs produced by Frank Gehry working with two senior designers are developed by a set of dedicated design teams. This design intent is developed primarily in a sculpted, physical model form. The "Master Model" surface geometries are created through acquisition of 3D geometrical information from the physical models by means of a 3D spatial scanner arm (Fig. 3). The scan process yields a constructive data point set from



Fig. 5. Quadrangular mesh of a double curved surface.

which then matching 3D computer geometry is developed. This ensures an accurate representation of the design intent within the computer, while at the same time allowing the data to be manipulated as required to yield constructible geometric solutions.

Once the framework for a given project is in place, in terms of programmatic requirements, form and layout, an initial computer model is built. The model is continuously refined during the design development process. Over time, the computer model acquires a large amount of information that makes the Master Model into an integrated, three-dimensional database



Fig. 4. The DG Bank Berlin atrium roof.



Fig. 6. Basic geometric principle for planar quadrangular mesh.



Fig. 7. Geometric principle for translation surfaces.

of project information that is considered the digital mockup of the project. For a more detailed discussion of the core technologies used at Gehry Partners, see [1] and [2].

### 3. The problem: free-form faceted glass structures

Gehry Partners and Schlaich Bergermann and Partner have successfully collaborated in the past on the solution of complex form glass roof systems, notably on the DG Bank building in Berlin (Fig. 4). On that project, the central atrium space was covered by a longitudinally symmetrical glass structure built of triangular planar glass facets. The adoption of this strategy for the MOT project proved unfeasible, mainly because of the sheer economy of scale generated by the quantity and dimensions of glass surface structures (roofs and some lateral enclosure elements). Although triangulated surfaces can describe any free-form shape [5], employed in construction, they are economically less advantageous than equivalent surface structures built of quadrilateral (four-sided) facets: Quadrangular mesh constructions require fewer machining operations on the glass, and fewer mullions (as they eliminate the diagonal mullion from one side of the triangle). However, this achieved economy of scale can only be maintained if the quadrilateral facets of the surface structure are maintained planar: the cost of single- or double-curved glass facets would immediately void the premise for a quadrilateral solution.

There are, however, geometric principles that can guarantee the geometric planarity of facets in a quadrangular mesh system, using translation surfaces. Gehry Partners leveraged Schlaich Bergermann & Partners' experience to use this method on the MOT project.

### 4. Translation surface structures: basic geometric principles

Quadrangular-meshed nets may be used to describe any double-curved surface, but usually the quadrangles of the surface are not planar (Fig. 5). In the following section, simple methods are presented for creating an almost unlimited multitude of shapes for double-curved surfaces with planar quadrangles.

One method is based on the simple principle that two spatial, parallel vectors are always defining a planar quadrangular surface. The vectors and the connection between their points of origin and end



Fig. 8. Translational surface covering an elliptical (top left) or circular plan.



Fig. 9. Joining of translational surfaces.

points make up the edge of the quadrangular surface. This is not the only method—but a rather simple one—because a planar surface may also be defined by two vectors not running parallel to each other.

Assuming one direction of the quadrangularmeshed net to be the sectional curve with its individual sections being the lateral edges, and assuming the other direction with its individual sections being the longitudinal edges and regarding the two lateral respective longitudinal edges as vectors, there results two design principles for plane quadrangular mesh:

- (a) The longitudinal edges of a row of mesh form parallel vectors (Fig. 6a).
- (b) The lateral edges of a row of mesh form parallel vectors (Fig. 6b).

## 4.1. The longitudinal edges of a row of mesh form parallel vectors

Any spatial curve may be chosen as sectional curve—it does not have to be plane. The parallel vectors generate the initial row of mesh (Fig. 6a). The

new sectional curve is the line between the end points of the vectors. The following row of mesh is generated according to the same principle, but of course the vectors here could have a different direction and length than the previous ones, thus adding one row to the next. Almost any shape consisting of trapezoidal mesh is possible. An analytical description of the resulting surfaces is omitted here because, following the procedure described above, these surfaces are easily generated with CAD.

To obtain homogeneous structures, analytical curves can be used to develop the sectional curves and the direction of the vector. If, for example, the sectional curve is plane and all vectors of the longitudinal edges have the same length, the result will be the design principle of the translation surface which plays a major role in the practical application.

### 4.2. A special application: the translation surface

As published already in [3,4,6], translation surfaces permit a vast multitude of shapes for grid shells consisting of quadrangular planar mesh.



Fig. 10. Hyperbolic paraboloid as translational surface (left) and joining possibilities (right).



Fig. 11. Translation surface covering two semi-circular plans connected with tangential edges.

Translating any spatial curve (generatrix) against another random spatial curve (directrix) will create a spatial surface consisting solely of planar quadrangular mesh (Fig. 7). Parallel vectors are the longitudinal and lateral edges. Subdividing the directrix and the generatrix equally results in a grid with constant bar length and planar mesh.

If, for example, one parabola (generatrix) translates against another parabola (directrix) perpendicular to it, the result will be an elliptical paraboloid with an elliptic layout curve. Two identical parabolas generate a rotational paraboloid with a circular layout curve (Fig. 8). Innovative shapes can be created by adding translation surfaces (Fig. 9).

A directrix curving anticlastically to the generatrix results in a hyperbolic paraboloid, which can also be formed by two systems of linear generatrices (Fig. 10). This allows for the creation of hypar-surfaces with straight edges which are easy to store and can be joined in a variety of combinations (Fig. 10). A layout of two circles connected by their tangents can be covered by a translation surface consisting of two rotational paraboloids with a hypar-surface in between (Fig. 11). All these examples consist of a grid with constant bar length and planar mesh.

This paper presents only the basic possibilities. A greater variety in shapes is described in [3].

# 4.3. The lateral edges of a row of a mesh form parallel vectors

Any—not necessarily linear—spatial curve can be selected as sectional curve. The new sectional curve will be generated by parallel translation of its lateral edges, resulting in a new vector of these edges. Its randomly chosen length determines the shape of the new sectional curve, which is not similar to the previous one (Fig. 12). The longitudinal edges of the plane quadrangular mesh are determined by the line between the points of origin and the end points of the respective lateral edge vectors.

The next row of mesh will be created following the same principle, with the shape of the new sectional curve depending on the length of the lateral edges. Thus, row after row is generated and any shape with plane trapezoidal mesh is possible.

To obtain homogeneous structures, the parallel vectors of the lateral edges can be developed from analytical curves by centric or eccentric expansion or by equidistant parallels. For example, the centric expansion of a sectional curve results in surfaces with homogeneous, longitudinal edges respective to tapering rows of mesh. These surfaces obtained by scaling a translation surface are called in this paper scale-trans-surfaces.



Fig. 12. Reduction of longitudinal edges by introducing a triangular element (right).



Fig. 13. Expanding a curve centrically or eccentrically results in parallel lateral edges.

To avoid extreme concentration, the number of longitudinal edges may be reduced by setting the vector length to zero, if necessary (Fig. 12).

### 4.4. A special application: scale-translation surfaces

The centric expansion of any sectional curve yields a new one with parallel edges (Fig. 13). The center of expansion may be chosen at random. Centric expansion causes each lateral edge vector of the sectional curve to lengthen or shorten by the same factor, while maintaining its direction. Thus, from the (n)th sectional curve evolves the similar (n+1)th sectional curve which is then spatially translated-although without rotation-to any given spot (Fig. 14). The longitudinal edges are determined by the line between the points of origin and the end points of the respective lateral edge vectors. The next row of mesh will be created following the same principle with the shape of the new sectional curve depending on the selected expansion factor. In this way row after row is generated.

To obtain homogeneous structures, analytical curves such as circles, ellipses, hyperbolas or poly-

nomials can be divided equally (identical lateral edge length) and expanded centrically. Translating the center of expansion of the resulting sectional curves along an analytical spatial curve (generatrix), and adjusting the translation distance to the expansion factor, results in homogeneous surfaces with longitudinal edges of the same length in each mesh and lateral edges of the same length in each sectional curve. Fig. 15 shows a double-curved surface with plane quadrangular mesh, created by centrically expanding elliptical curves and translating the center of expansion along a spatially curved directrix. Fig. 16 describes a sectional curve consisting of two adjoining elliptical curves. The sectional curve in Fig. 17 is composed of randomly chosen curves.

The "trajectory dome" in Fig. 18 is created by centric expansion of a spatially curved generatrix and translation along a defined edge curve. The result is a dome with plane quadrangular mesh over a square plan! Sectional curves evolving from a circle with the center of expansion in the circle's center and are then translated along a straight directrix create a special application, the rotational surface (Fig. 19). Translat-



Fig. 14. Surfaces with plane quadrangular meshes are generated by expansion and translation.



Fig. 15. Surface composed of plane quadrangular mesh generated by centrical expansion and translation (scale-trans surfaces).

ing these curves along a curved directrix yields a totally different surface with plane quadrangular mesh. This illustrates the multitude of shapes that are possible with this procedure.

# 4.5. A special application: equidistant sectional curves

In some instances, it might be more advantageous to generate the new sectional curve by equally translating the individual lateral edges instead of using centric expansion. In this case the points of origin and the end points of the lateral edge vectors are on the bisector of the plane sectional curve (Fig. 20). Again, the lines between the points of origin and the end points of the respective lateral edge vector produce the longitudinal edges. The next row of mesh will be created following the same principle. The resulting sectional curves may be translated at random or along an analytical spatial curve (directrix).

### 4.6. Reference projects by Schlaich Bergermann & Partners

The principle of an evenly-meshed translation surface with different parabolas as generatrix and directrix was first realized with the roof over a courtyard in Rostock, Germany (Fig. 21). A parabola (generatrix) translating across another parabola (directrix) perpendicular to it results in an elliptic layout curve and an evenly meshed net consisting of plane quadrangular mesh.



Fig. 16. Expansion of an elliptical curve and translation of the curve along a spatially curved directrix.



Fig. 17. Expansion of a random curve and vertical translation of the curve.

In 1999, a sequence of grid domes was chosen for the fair in Hannover, Germany (west entrance). Generatrix and directrix are identical parabolas (Fig. 22), resulting in domes over quadrangular layouts that can be linked at random, again having quadrangular or rectangular layouts.

One example of designing rather difficult geometries as translational surfaces is the roof over a courtyard in Stuttgart, Germany. Although in this case there are several adjoining irregular courtyards, it was possible to create a continuously curved transition area consisting solely of plane and evenly meshed quadrangles (Fig. 23).

Trapezoidal layout of courtyards such as in Leipzig, Germany (Fig. 24) can be roofed with a hyperbolic paraboloid generated by anticlastic parabolas as generatrix and directrix.

For the hippopotamus house in the Berlin Zoo, two parabolas with a freely defined transition curves were chosen as a directrix for the roof over two circular ponds (Figs. 25 and 26).

The plan of the glass roof over the platform of the new Lehrter railroad station in Berlin follows strictly the flaring tracks. The three-centered arch crosssection of the roof, as shown in Fig. 27, allows for optimal adjustment to the clearance and to the flaring tracks. To obtain planar quadrangular mesh, the sectional curve was determined by centric expansion. Thus the rise of the hall and the roof's profile increase with the flaring, an effect that enhances the design.

### 5. Implementation

#### 5.1. Parametric associative modeling

The beginning of the Design Development phase of the MOT project coincided with the introduction at Gehry Partners of the new CATIA platform, Version 5 (currently Release 8). CATIA V5 differs from the previous V4 in a great number of ways, presenting improvements in ease of use, interactivity and compatibility. The greatest difference is that V5 is a fully parametric associative modeling system, and can be greatly controlled by harnessing its associative capa-



Fig. 18. "Trajectory dome" covering a square plan with plane quadrangular mesh.



Fig. 19. Rotational surfaces as a special application of centrically expanded circular curves.

bilities by means of built-in scriptable intelligence features known as KnowledgeWare.

This coincidence has proven fortuitous, because V5 has allowed us to program the necessary rules and constraints into CATIA that are necessary to generate translation surfaces that will guarantee planar, quadrilateral facets. Programming here is not intended literally as 'coding instructions', though the result is the same. Rather it is the ability to embed logical constraints, directives and effectors into the model while it is being built. Consequently, any changes made on the completed model are only possible within the framework of these constraints that control the "translationalness" of the surface structure: in other words, any shape possible that can be generated by the constrained model is guaranteed to be a translation surface because of the rules and constraints that control it. In short, programming in the sense of CATIA V5's capabilities can be understood as a means of imbuing a design with control features that correspond, conceptually, to an implementation of the notion of spatial programming or design structuring.

### 5.2. Structuring of the model

The sequence of creating such models has been relatively conceptually straightforward, though navi-



Fig. 20. In the case of equidistant curves the lateral edges meet in the bisector.

gating the meanders of CATIA V5's complex feature set and associative hierarchy system has proven a challenge at times. Because the system records every element created as part of a semantic logical specification tree, it is possible to continue referencing and controlling each geometric element of the model as it is developed. In other words, geometry within the model is constructed as a function of other pieces of geometry previously placed, with the associative relation being maintained in the specification tree. Any changes thus made to geometric elements upstream in the tree immediately trigger an 'update' or re-configuration in those derived downstream. Furthermore, it is possible to embed into the model numerical and logical parameter values that can be used to control the geometry, and can be linked to the knowledge features described above to produce a reconfigurable intelligent model that permits controlled manipulation of the geometry within the constraint sets placed on it.

For the MOT glass roof structures, we used the scaled translation surface structure described in Section 4.4 in order to be able to match the translation surface as closely as possible to the original designed shape. The resulting parametric model was in fact relatively straightforward. It was built in the same logical sequence used to geometrically determine the translation surface geometry:

- I. Three sets of control points are created for three control curves:
  - a. Generatrix
  - b. Directrix
  - c. Scaling curve (Law)

Curve (c) is a spline curve used to uniformly control the scaling in the scaled translation surface. CATIA can read numeric values off curves



Fig. 21. Rostocker Hof, Rostock, Germany; grid dome as translation surface.

as a 'law' function that can be used, as parameters, to drive other functions—in the case the scaling of the translated curves over the directrix.

- II. The generatrix, directrix and law curves are created over the control point sets.
- III. Two distance parameters G and D are used to control the spacing of vertices in the U and V directions on the translation surface.
- IV. Equidistant points are placed on the directrix at D distance intervals using Euclidean (spherical absolute) spacing to ensure equal length for each mullion line.
- V. The generatrix is translated over the equidistant points on the directrix; at each translation, the generatrix curve is also scaled by a value S read

from the law curve at the correspondent abscissa value on the length of the law curve reference.

VI. Equidistant points are placed on the translated and scaled generatrix curves at (G\*S) scaled distance intervals using Euclidean (spherical absolute) spacing to ensure equal length for each mullion line on each specific generatrix curve.

### 5.3. Control of the translation surface structure

Control of the surface is achieved by combining a combination of any of the following:

I. Modifying the control points for the directrix curve.



Fig. 22. West entrance, Hannover Fair, joined translational surfaces.



Fig. 23. Courtyard roof of the former Bosch Area, Stuttgart, Germany; grid dome as translational surface with planar mesh.

- II. Modifying the control points for the generatrix curve.
- III. Modifying the control points for the law (scaling) curve.
- IV. Modifying the D and G distance parameter values.

We now have a constrained reconfigurable model of the translation surface structure. By modifying individual or combinations of the above control elements, we can reconfigure the model to match the shape of the original scanned shape (see Section 5.5). The generated surface is then trimmed to match the plan projection of the original design shape (Fig. 28). Because of the constraints of the translation surface, it is not possible to perfectly match the original surface, but the combination of translation and scaling gets quite close.

Once the shape of the surface has been established, the entire construction of the glass structure can be placed on it. The generated geometry permits the spatially correct placement of mullion elements on the underlying wireframe (Fig. 29). Due to the associative nature of the model, any changes made "late in the game" to the shape of the surface for aesthetic, structural, or other reasons will percolate through the



Fig. 24. Courtyard roof industriepalast, Leipzig, Germany.



Fig. 25. Glass dome of the Hippo House at the Berlin Zoo as translation surface with planar quadrangular mesh.

model and reflect through to the final surface, including mullion layouts.

#### 5.4. Automation and geometric correctness

The generation of the geometric and topological requirements for a translation surface structure can be computationally automated such that, by selecting the appropriate input elements (normally the control elements listed in Section 5.3), the system can automatically generate the entire parametric structure which, by definition, guarantees true translation surface conformity. One can conceive of this approach as a kind of automated 'translation surface template'.

### 5.5. Form-finding processes

As we have seen, we can structure a reconfigurable model that contains, by its definition, the rules and constraints that make a translation surface. In other words, once the structure is in place, it still must be 'shaped', using the control elements described in Section 5.3 to approximate as closely as possible the original surface created in model form by the designers.

This form-finding process itself presents a number of options, depending on where one sees the actual design event occurring. If we assume that the design of the shape in question has been completed in model



Fig. 26. Glass dome of the Hippo House at the Berlin Zoo.



Fig. 27. Platform roof of the Lehrter railroad station in Berlin. Plane quadrangular glass panels obtained by centric expansion and translation of the sectional curves.

form, then the entire process described in this paper is nothing other than a mechanistic solution for the construction of said shape. However, because a design process is in fact more complex and cyclical, then one could assume that even after the translation structure has been parametrically modeled, modifications can still occur that can substantially affect the actual form.

In the Gehry studio, revisions are constantly made to the designs in physical model form, either for aesthetic, programmatic or—in the case of the glass structures—for structural reasons. For example, substantial modifications to the designers' approach in determining their shape had to be made, in order to ensure greater degrees of curvature in the surface that would ensure structural feasibility without, say, the need for external support elements such as columns.

*Manual form-finding*: The directrix and generatrix curves can be modified to 'tweak' the surface into conforming to the original, as can the scaling 'law' curve. Interestingly, we have found that, to a very high



Fig. 28. Parametric-associative model showing original scanned surface from physical model (dark blue), generatrix and directrix curves, and trimmed generated translation surface (violet).



Fig. 29. Mullions placed on the wireframe of generated translation lines are positioned correctly in space to guarantee planar quadrilateral facets. Any change in the underlying surface automatically updates the mullions' positions.



Fig. 30. Optimization techniques can be used to approximate the form of the generated translation surface (yellow) to that of the original scanned surface (blue) by minimizing the distance between corresponding sampled surface points.

degree, human intuition is a substantial factor in this method. Determining, for example, the starting configurations for directrix and generatrix in such a way that they produce very close configuration from the outset is a qualitative judgment that comes as much from experience with the project in hand as it does from the user's CAD modeling abilities.

One scenario that has been explored at Gehry Partners is the designer working directly on the parametric model with the CAD operator: the designer can make changes 'live' to the model. Comments such as 'can you make it a little more curved here' or 'flatter there' can be translated directly into the structure's form by manipulating the control elements on the fly.

*Computational form-finding*: If we assume that no modifications are made to the model's shape, then one could enable the computer to automatically approximate, as closely as possible, the translation structure to the original form (Fig. 30). This can be done through the application of search methods, such as Simulated Annealing or Linear Hill-climbing, or non-linear techniques such as Genetic Algorithms. Regardless of the method, the search mechanism continues to modify the control parameters while measuring the resulting conformity of the surface to the original design shape.

### 6. Conclusions

Using CATIA Version 5 enabled us to clarify the procedures necessary for implementing free-form panelized structures using translation surface geometries. Parametric/associative infrastructure, combined with constraint-solving capabilities, allows for the structuring of design concepts early; as well as for controlling design development tool further downstream. Furthermore, by combining the techniques described in Sections 5.4 and 5.5, we can see the parametric model as a kind of machine that is capable of producing translation surface structures in response to a particular set of inputs.

In conclusion, it could be stated that parametric associative models are also a design refinement tool as well as a design development and production tool. By embedding logical control features and constraints into the model, and using its capabilities for controlled user interaction, the user can take advantage of the parametric and associative features of the model to refine a design within the context of the logic structured within the model.

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