FOLDABLE / UNFOLDABLE CURVED TENSEGRITY SYSTEMS
BY FINITE MECHANISM ACTIVATION

Ali SMAILI¹, René MOTRO²

¹Laboratoire de Structures Légères pour l'Architecture, Ecole Nationale Supérieure d'Architecture de Montpellier, 179 rue de l'Espérou 34093 Montpellier cedex 5, France, smaili.ali@caramail.com
²Laboratoire de Mécanique et Génie-Civil - UMR5508, Université de Montpellier II, cc048, Place E. Bataillon, 34096 Montpellier, France, motro@lmgc.univ-montp2.fr

Editor’s Note: This paper is the winner of the 2006 Tsuboi Award for the outstanding paper published in the proceedings of the annual IASS Symposium. It is republished here with the permission of the editors of the proceedings of the IASS 2006 Symposium “New Olympics, New Shell and Spatial Structures”, held in October 2006 in Beijing, China.

ABSTRACT

The purpose of this paper is to present a complete design study of foldable and unfoldable curved tensegrity systems with all the involved steps: generation of the curved tensegrity truss, implementation of finite mechanisms, structural analysis and folding/unfolding. Three criteria are required to introduce the finite mechanisms and self stress saving throughout the folding procedure. The new tensegrity configurations which meet these criteria are classified and illustrated with respect to three parameters: direction of truss beams, surface's mean curvatures and direction of articulation axes. Several configurations are then presented.

Keywords: Double Curved Tensegrity systems, Folding/Unfolding, Finite mechanisms, Self-Stress.

1. INTRODUCTION

The study of foldable/unfoldable tensegrity systems with different geometry implies a clarification of the relationship between form, self-stress and finite mechanisms. The geometry of the systems and the state of self-stress depend on the kinematics of the elements during the folding process. In tensegrity systems these obvious relationships become increasingly complex when other aspects are associated with the classical ones: additional requirement of a physical model, manufacture and function constraints.

Double curved tensegrity systems may offer efficient structures if they are developed with form and structure, function and economy kept in mind. However, not every form is susceptible to be chosen. Double curved surfaces alone cannot guarantee an executable design which satisfies the folding process requirements (geometrical and kinematical constraints) [1]. The research, which is described in this paper, aims to make folding and deployment of double curved tensegrity grids more accessible and focuses on the following objectives:

- Research of folding and deployment procedures of tensegrity systems without the necessity to remove completely the self-stress.
- Application to several structural configurations, and in particular for double-curved tensegrity grids.

Figure 1: Double layer tensegrity grid.(Tensarch project)
2. PROBLEM AND AIMS

The main problem in the present design of tensegrity structures is to find a self-stress equilibrium configuration throughout the folding process. This is an additional difficulty in comparison with traditional folding and deployment of tensegrity systems [2]. However, the crucial design step is described by the creation of mechanisms that enable reliable folding/unfolding.

In the field of deployable structures we published recently a paper [3] on folding tensegrity systems like the grid of Figure 1 (with the same topology of members). The self-stress is maintained throughout the folding process for significant parts of the structure. This study has enabled us to generalize this process to other tensegrity grids with total Gaussian curvature null, positive or negative.

In order to keep self-stress states, it is necessary to choose systems with specific geometrical characteristics and simultaneously with ability to be folded.

3. TYPOLORIES

According to the multiplicity of parameters which can be taken in account, only a few number of typologies will be considered, mainly the periodic ones.

The new considered tensegrity configurations are designed with the same basic concept which was used for the flat grid, that the 2V expander. The 2V expander is the key point for self-stress implementation [3]. They are classified according to three geometrical parameters. Each of them is declined in sub-parameters the variations of which give access to an infinity of configurations. We use the following parameters:

1. Direction of ‘truss beams’
   - Systems with parallel ‘truss beams’.
   - Systems with ‘truss beams’ crossing along one or several axes.

2. Surface's mean curvatures
   - Surfaces with identically null total curvature.
   - Surfaces with positive total curvature
   - Surfaces with negative total curvature

3. Direction of articulation axes
   - Systems with parallel articulation axes
   - Systems with converging articulation axes.

For every configuration, the system description contains an explanation of the geometrical constraints that allow a complete folding/unfolding (in the final position, all components are ideally contained on the folding plane.) and the different configurations from initial to final position.

In this paper, we focused our presentation on configurations which have parallel ‘truss beams’ and parallel articulation axes, and total curvature identically null, positive or negative total curvature. These choices allow us to distinguish four different typologies:

I. Planar tensegrity system.

II. Double curvature tensegrity system I (surface with total curvature identically null).

III. Double curvature tensegrity system II (surface with positive total curvature).

IV. Double curvature tensegrity system III (surface with negative total curvature).

3.1. PLANAR TENSEGRITY SYSTEM

This configuration has been illustrated and established in our previous paper published in IASS journal [3]. The constitutive modules are distributed according to a regular grid (Figure 2a). In order to end with a configuration completely mapped on vertical folding plane, the elementary module has to respect the following geometrical conditions (Figure 2b):

- The vertical axes AA', BB', CC' and DD' are parallel
- The faces AA'BB' and CC'DD' are parallel and identical
- The faces AA'CC' and BB'DD' are parallel and identical

This elementary observation leads to two possible finite mechanisms but we are interested only in one of them. The horizontal mechanism is a movement by “shear action”, keeping horizontal the quadrangular face A'B'C'D'.
The target folding plane is defined by the square AA'BB' (Figure 2b). All the lateral faces are not distorted during the displacement induced by the mechanism, when the self-stress is maintained.

Kinematic properties of the system are illustrated in Figure 3, containing numerous finite mechanisms the direction of which varies along with the horizontal plane. One of these finite mechanisms
can end on a vertical folding plane, which corresponds to one of four vertical lateral boundaries of the grid. The faces which are initially parallel to the folding plane get closer and closer together, following a zigzag path. In the final position, all modules are mapped on the folding plane (Figure 4b).

3.11. DOUBLE CURVED TENSEGRITY SYSTEM (SURFACE WITH TOTAL CURVATURE IDENTICALLY NULL)

This folding process can be applied to a curved grid using the same implementation of finite mechanisms. The new double curved grid itself has of course a consequence on the resulting position, and simultaneously on the number and the value of self-stress.

The first problem is to find a method which allows the generation of new configurations with double curvature, starting from flat configurations. The second difficulty in the design of curved tensegrity system is to find a self-stress equilibrium configuration. Several methods are available, but a single method is suitable for general problems. Here, an approach, involving an orthogonal projection method, is proposed.

For a double curvature tensegrity system, this procedure is illustrated in the Figure 5a. The process implies the projection of a planar grid, which has been described in the preceding paragraph, on the target geometry, a surface with total identically null curvature (cylinder) (Figure 5). To preserve the folding compatibility in this case, the direction of projection is the direction of articulation axes. Thus, the generation process implies the following steps:

- Defining the target geometry (general shape, size, curvature radius, etc…) knowing that it has to be in accordance with the original typology.

**Figure 4:** Folding of a real physical model by flattening the truss beams.

**Figure 5:** Orthogonal projection method for double curved tensegrity grid.
• Definition of the intersection points between the projection axes and target geometry.

• Folding/unfolding the new geometry according to the mechanisms which are linked to the main orthogonal directions of the grid.

Finally, the projected geometry which is based on a simple pavement is characterized by different lengths for cables and struts when compared with its ‘mother’ geometry. But, the projection principle has preserved the direction and the number of finite mechanisms (Figure 6c).

To fold the new system, the method consists in flattening the parallel modules on the folding plane. The vertical edges of the parallel faces are the rotation axes. A force is applied on external node along the folding direction in order to activate a shear mechanism; the system is reticulated and the degrees of freedom are fixed for ‘half’ of the system Figure 6. During this shear process all lateral faces move following a zigzag path. The result of the virtual simulation of the mechanism is given in the Figure 6c.

3.III. DOUBLE CURVED TENSEGRITY SYSTEMS (SURFACES WITH POSITIVE TOTAL CURVATURE)

The configuration here considered is illustrated in the Figure 7a. The method considered here was described in the previous paragraph. Starting from the ‘mother’ geometry, the configuration is generated by the orthogonal projection on the target surface, with a positive total curvature. The direction of projection has to be parallel to articulation axes of the ‘mother’ geometry.

The intersection of four projection lines with the surface defines a module whose typology is identical to the mother elementary module. Cinematically, this new module has also horizontal finite mechanisms. Geometrically, the new geometry of the system is characterized by the direction of projection and the curvature radii of the surfaces. These are the parameters of the evolution. Mechanically, in the initial configuration (planar grid), the system has one or more states of self stress (the number of state of self stress is up to the typology of the system, geometry, fixed nodes…). But, the new configuration has also different states of self stress, which are linked to the geometry.

![Figure 6: Foldable double-curved tensegrity grid process (surface with total curvature identically null).](image)

![Figure 7: Orthogonal projection method for double-curved tensegrity grid (with positive total curvature).](image)
The juxtaposition of several new modules with a common vertical direction gives a sector the geometrical compatibility of which allows a zigzag path trajectory which leads the structure to be mapped on a one of two orthogonal vertical planes. The folding method described in the preceding paragraph leads to the development of a folding process which can be applied to a curved grid. The mechanism activation consists to apply actions, which are orthogonal to the folding plane: at the end of the movement all the constitutive modules are mapped on the folding plane Figure 8.

![Figure 8: Foldable double-curved tensegrity grid process (surface with positive total curvature).](image)

3.IV. DOUBLE CURVED TENSEGRITY LATTICE (SURFACE WITH NEGATIVE TOTAL CURVATURE)

To generate a foldable/deployable tensegrity system with two layers resting on two ‘parallel’ surfaces with negative total curvature, we proceed in accordance with the method, which has been established in the preceding paragraphs. The geometric configuration created in Figure 9 is a grid generated by an orthogonal projection of the same flat grid, as in preceding typologies. Since the grid is made up with the same principle, the folding process, as illustrated before, should be activated by forces which create a “shear” mechanism. This method is also a folding process ending on a vertical plane: convex or concave beams move following a zigzag path (Figure 10).

![Figure 9: Orthogonal projection method for double-curved tensegrity grid (with negative total curvature).](image)

![Figure 10: Foldable double-curved tensegrity grid process (surface with negative total curvature).](image)
4. FOLDABLE/UNFOLDABLE DOUBLE CURVED TENSEGRITY LATTICE ANALYSIS

Four tensegrity grids, a flat grid and three double curved grids were examined (Figure 11). In his doctoral thesis [3], Smaili described the structural behavior of the first grid, during the folding process. For this description, a numerical modeling called ‘tensegrity2000’ is used (it has been developed in the laboratory of mechanics and civil engineering).

![Computer modeling of four tensegrity grids](Figure 11)

Each system is made of 12 struts and 28 cables according to the typology of 2V spacer (Figure 11). When the grids were self-stressed, a force is applied on a peripheral joint for each system. This force must be orthogonal to folding plane in order to activate a shear mechanism. Some other nodes are fixed in order to constraint the displacements and to maintain the folding plane in a stable position. During the folding process, closer examination revealed that the number of states of self-stress is not modified, but their level is modified. The four systems, which were studied, could be folded in a satisfactory way. However, high local stresses were induced, and specifically on vertical cables (Figure 12). Consequently further studies are required to minimize these local stresses.

![Variations in the degree of self-stress throughout the folding process](Figure 12)

### Figure 11: computer modeling of four tensegrity grids, (a) grid plane, (b) cylindrical grid, (c) spherical grid, (d) parabolic-hyperbolical grid.

### Figure 12 (a) y (b): Variations in the degree of self-stress throughout the folding process, (a) according to grid plane, (b) according to cylindrical grid.
systems and the state of self-stress depend on the movability of the elements during the folding process. In tensegrity systems these classical relationships become increasingly complex when the aspects of physical model, manufacture and function are added.

We have established new configurations, covering a wide range of possibilities for folding double curved (identically null, negative and positive) tensegrity system. In each case, during the folding process, some parts of the tensegrity systems remain in a state of self-stress. The parameters' diversity is very large. Preliminary results regarding their mechanical behavior, the evolution of the self-stress and the finite mechanisms exhibit some stress increasing which have to be carefully studied.

5. CONCLUSION

Folding and unfolding tensegrity systems with distinct geometries have been described, with emphasis on the relationship between form, self-stress and finite mechanisms. The geometry of the

REFERENCE

